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A CONTINUOUS CORRELATION EQUATION FOR HEAT TRANSFER FROM CYLINDERS TO AIR IN CROSSFLOW FOR REYNOLDS NUMBERS FROM 10^{-2} TO 2×10^5

R. M. FAND and K. K. KESWANI
Department of Mechanical Engineering, University of Hawaii, Honolulu, Hawaii, U.S.A.

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STATEMENT OF OBJECTIVE

THERE exist today reliable experimental data for heat transfer by forced convection from circular cylinders to air in crossflow for Reynolds numbers from 0·01 to 2×10^5. Heretofore, in order to correlate this extensive body of data, it has been found necessary to subdivide the overall range of the Reynolds number into five or six sub-intervals and to develop a family of five or six corresponding correlation equations. The objective of the investigation reported herein was to develop a single, continuous, algebraic correlation equation for heat transfer from circular cylinders to air in crossflow for the entire range of Reynolds number for which reliable data are available.

PHYSICAL BASIS AND GENERAL FORM OF THE CORRELATION EQUATION

In a previous publication Fand [1] demonstrated that heat transfer from cylinders to *water* in crossflow can be correlated by means of an equation having the following form:

$$Nu_f Pr_f^{-0\cdot3} = C_0 + C_1 Re_f^{0\cdot5} + C_2 Re_f^n \qquad (1)$$

where the C's and n are constants, and the subscript f indicates that fluid properties are evaluated at the mean film temperature. In this equation the term $C_1 Re_f^{0\cdot5}$ represents convective heat transfer to the laminar boundary layer on the front portion of the cylinder, and the term $C_2 Re^n$ accounts for heat transfer by convection from the rear portion of the cylinder, where separation occurs. The factor $Pr_f^{-0\cdot3}$ in equation (1) accounts for variations in the properties of liquids. Collis and Williams [2] have found that for air the analogous factor is $\mathscr{T} = (T_f/T_\infty)^{-0\cdot17}$, where T_f and T_∞ are the mean film and ambient absolute temperatures, respectively. The correlation equation adopted here is the same as equation (1) except that $Pr_f^{-0\cdot3}$ is replaced by $\mathscr{T}$ and the constant n is replaced by the variable $x = f(Re_f)$: thus

$$Nu_f \mathscr{T} = C_0 + C_1 Re_f^{0\cdot5} + C_2 Re_f^x. \qquad (2)$$

The constants C_0, C_1, C_2 and the variable exponent x in equation (2) are to be determined from the analysis of experimental data. The constant C_2 and the variable exponent x may be written as follows: $C_2 = C_1 C_3$ where C_3 is another constant, and $x = 0{\cdot}5 + y$. Substitution in equation (2) yields

$$Nu_f \mathscr{T} = C_0 + C_1 Re_f^{0\cdot5} + C_1 Re_f^{0\cdot5}(C_3 Re_f^y), \qquad (3)$$

$$Nu_f \mathscr{T} = C_0 + C_1 Re_f^{0\cdot5} + C_2 Re_f^{0\cdot5+y}. \qquad (4)$$

Equations (3) and (4) amount to a physical statement concerning the relationship between the convective heat-transfer rates from the front and rear portions of a cylinder in crossflow; namely, that the convective heat-transfer rate at the rear is equal to a *fraction* of the convective heat-transfer rate at the front and this fraction is $C_3 Re_f^y$.

REVIEW OF EXPERIMENTAL DATA

The correlation equation developed herein is based upon experimental data contained in publications by Hilpert [3], King [4] and Collis and Williams [2]. Hilpert's data were obtained by exposing a series of twelve heated cylindrical specimens to air in crossflow in the range $2 < Re < 2{\cdot}3 \times 10^5$. Hilpert correlated his experimental data (excluding those for Specimen No. 5, which are inconsistent with the rest) by dividing his overall range of Reynolds number into five intervals in each of which the data were represented by a function of the type

$$Nu_i = CR_i^n. \qquad (5)$$

In equation (5) the subscript i refers to integrated mean values of fluid properties. The results of Hilpert's measurements (excluding Specimen No. 5) are plotted in Fig. 1 in terms of $Nu_f \mathscr{T}$ and Re_f.

In the range of Reynolds numbers less than about 100, which is of paramount importance in hot wire anemometry, the work of King [4] has been regarded for many years as the primary source of reliable experimental data. However,

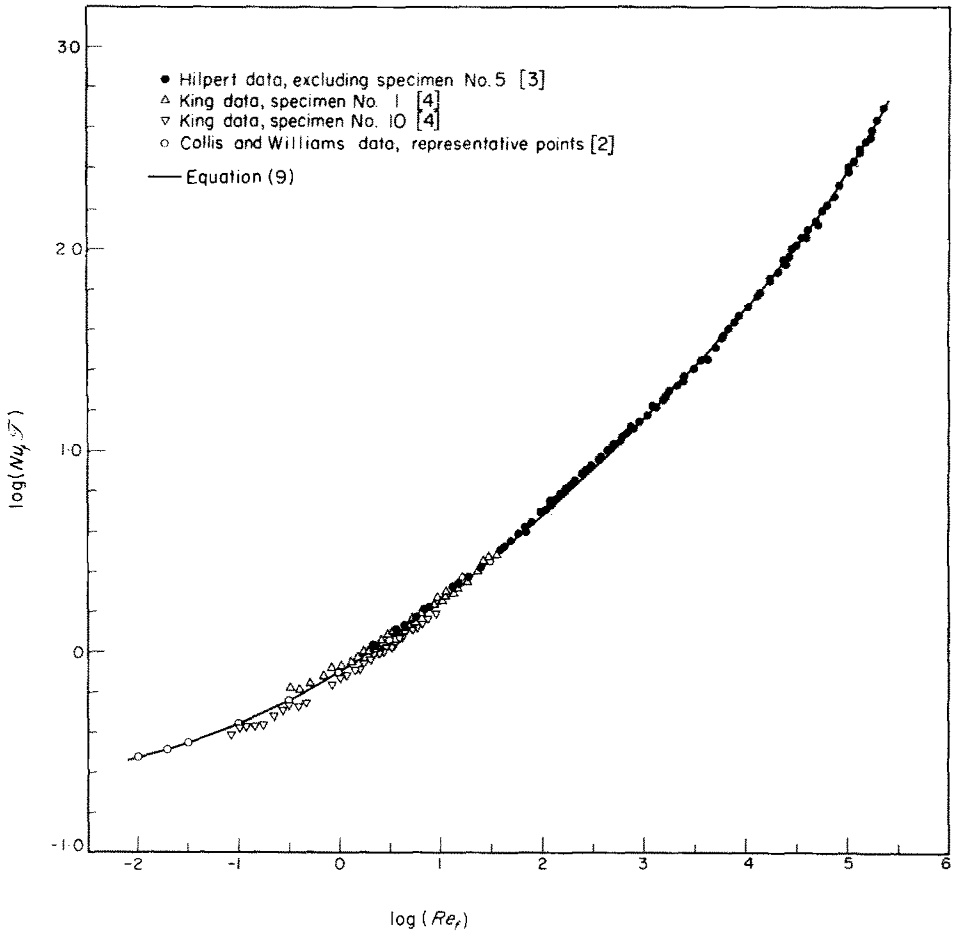

FIG. 1. Plot of experimental data and correlation equation.

in 1959 Collis and Williams [2] published experimental data which are generally regarded to be the most accurate low Reynolds number data that are available today. These data, which lie in the range $0.01 < Re < 140$, exhibit less scatter than do those of King, probably due to the fact that King mounted his heat transfer specimens (fine wires) on a whirling cantilevered arm, whereas Collis and Williams performed their experiments in the more controlled environment of a low-turbulence wind tunnel. Collis and Williams presented their experimental results in the form of a graph. The coordinates of representative points of this graph are plotted in Fig. 1.

Collis and Williams correlated their data in the range $0.02 < Re_f < 140$ by the following piecewise continuous function:

$$\begin{aligned} Nu_f \mathcal{T} &= 0.24 + 0.56\, Re_f^{0.4} \quad &\text{for } 0.02 < Re_f < 44 \\ Nu_f \mathcal{T} &= 0.48\, Re_f^{0.51} \quad &\text{for } 44 < Re_f < 140. \end{aligned} \tag{6}$$

They state that, for $Re_f = 0.02$, equation (6) overestimates the Nusselt number by 2 or 3 per cent, and the error increases rapidly if the same relation is used at smaller Reynolds numbers. Therefore, for values of Re_f between 0.01 and 0.5, they correlated their data by a different equation, namely,

$$Nu_f \mathcal{T} = \frac{1}{1.18 - 1.10 \log (Re_f)}. \tag{7}$$

The data of King were taken with a series of ten wires in the range of Reynolds number from approximately 0.1 to 50. The data for Wire Nos. 1 and 10 are plotted in Fig. 1 (excluding 5 points not consistent with the rest).

CORRELATION AND DISCUSSION

The function y in equation (4) was assumed* to have the following form

$$y = M + NRe_f^s. \tag{8}$$

The optimum values of the constants C_0, C_1, C_2, M, N and s were determined to be 0·184, 0·324, 0·291, − 0·253, 0·0407 and 0·168, respectively. With these constants, the final correlation equation becomes:

$$Nu_f\mathscr{T}\ 0{\cdot}184 + 0{\cdot}324\, Re_f^{0\cdot 5} + 0{\cdot}291\, Re_f^{\chi}$$
$$\text{where } \chi = 0{\cdot}247 + 0{\cdot}0407\, Re_f^{0\cdot 168} \tag{9}$$

A plot of equation (9) is shown in Fig. 1. The mean value and standard deviation of the error incurred by testing equation (9) against all the data points plotted in Fig. 1 are 0·37 and 5·52 per cent, respectively. These errors are negligibly different from the errors incurred by applying Hilpert's piecewise correlation technique. It is concluded, therefore, that equation (9) represents the experimental data with sufficient accuracy to render it useful. However, it should be noted that equation (9) represents the experimental data with greater fidelity in certain regions than it does in others. Thus, it correlates the data of Collis and Williams with less than 2 per cent error in the region $0{\cdot}01 < Re_f < 40$, but the error is consistently negative (as much as − 3·48 per cent) in the region $1{\cdot}5 < \log Re_f < 3$. The reason for this consistently negative error is, as may be seen in Fig. 1, that the data in this region fall on a nearly perfectly straight line segment—such a segment cannot be represented exactly by an equation having the form adopted here for correlation purposes. It is pertinent to mention that Hilpert did not make measurements of the intensity of turbulence in the air-stream incident to his test specimens. Kestin and Maeder [5] have estimated that the intensity of turbulence in Hilpert's experiments was approximately 0·9 per cent—such a level of turbulence can affect experimental heat transfer results by more than the difference between equation (9) and the data in Fig. 1.

One of the basic considerations that Fand used to develop equation (1) consisted of a set of known, experimentally determined values of the ratio, $\mathscr{R}$, defined as follows:

$$\mathscr{R} = \frac{\text{heat transfer from the rear portion of a cylinder}}{\text{heat transfer from the total surface of the cylinder}}.$$

Since the structure of equation (9) is based upon this same physical consideration, it is appropriate to calculate $\mathscr{R}$ by means of equation (9) and to compare these calculated values with corresponding experimentally determined values of $\mathscr{R}$. Such a comparison will reveal to what extent equation (9) conforms with the physical model upon which the whole

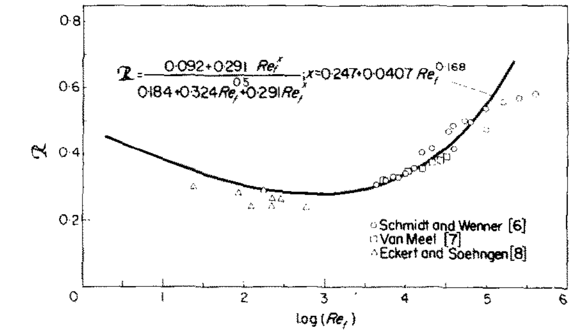

FIG. 2. Ratio of heat transfer from the rear portion of a cylinder to heat transfer from the total surface of a cylinder in crossflow of air.

correlation procedure depends. To this end, various sources [6–8] were culled for experimentally measured values of $\mathscr{R}$—these values are plotted in Fig. 2 together with the algebraic expression for $\mathscr{R}$ derived from equation (9). The quantity $\frac{1}{2}\,C_0 = 0{\cdot}092$ in the numerator of $\mathscr{R}$ accounts for half of the total heat transfer from cylinder when $Re_f \to 0$. The curve for $\mathscr{R}$ is in fairly good agreement with the experimental data.

CONCLUSION

A single, continuous algebraic equation has been determined which correlates heat transfer from cylinders to air in crossflow in the range $10^{-2} < Re < 2 \times 10^5$. It is anticipated that this correlation equation will have application in the field of hot wire anemometry.

ACKNOWLEDGEMENT

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* The validity of this assumption can be assessed only by evaluating the accuracy with which the final correlation equation represents the experimental data.

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FRANZ GRASHOF AND THE GRASHOF NUMBER

C. J. SANDERS and J. P. HOLMAN
Southern Methodist University, Dallas, Texas, U.S.A.
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THE GRASHOF number is widely used in all types of free convection problems, but unlike other dimensionless groups in the fields of heat transfer and fluid mechanics, the man for whom the grouping was named is not familiar to workers in the field. Consequently, the historical and biographical information which follows is of interest in establishing proper links with the past.

Jakob [1] attributes the naming of the dimensionless group

$$Gr = \frac{\rho^2 g \beta \Delta T}{\mu^2}$$

to Groeber [2] but points out that Groeber gave no reason for selecting the name "Grashof number". A careful check of the free convection literature before and after 1921 indicates that Groeber did indeed coin the phrase because the grouping was nameless before that time. Even Davis [3] still left the group without name in 1922 but it is likely that he did not have access to Groeber's text. By the time that Colburn's [4] excellent historical discussion of convection heat transfer correlations appeared in 1933, the name Grashof number was well established but the man for whom the group was named was still a mystery, at least in the heat transfer literature.

Although we cannot be sure, it appears that the dimensionless group was named for Franz Grashof (1826–1893), a very famous German engineer in his time. A search of the biographical literature reveals no other person named Grashof who could be a candidate for the naming. The biographical information which follows is a translation of the information available from Poggendorff's [5] and Matschoss [6].

Franz Grashof was born July 11, 1826 in Dusseldorf, and died October 26, 1893 in Karlsruhe. His grandfather experienced the overthrow and rise of Prussia as a school master in Prenzlau, and in 1813, at the age of 43, he took the field as a second lieutenant in the militia along with his students. After the end of the war he was appointed provisional director of public instruction on the lower Rhine. Grashof's father taught ancient languages as senior assistant master at the Royal Classical School in Dusseldorf. His son, Franz Grashof, was attracted early to technical subjects in contrast to the family tradition in the pure arts. At 15 years of age he left school to work as a mechanic and then attended trade school in Hage and secondary school in Dusseldorf.

From 1844 until 1847 Franz Grashof studied mathematics, physics and machine design at the Berlin Royal Technical Institute. During the German peoples movement of 1848 he served in the military in Dusseldorf. As a result of this experience he decided to join the German navy. He enlisted on the sailing ship "Esmeralda" as a seaman and made a voyage which lasted over $2\frac{1}{2}$ years and took him as far as the Dutch Indies and Australia. On the voyage he realized, however, that he was not destined for a mere practical vocation. After a short stay at home he returned to Berlin in 1852 and continued his studies. In addition, he was assigned to conduct a lecture on applied mathematics at the school. In 1854 he became employed as a teacher of mathematics and mechanics at the Royal Technical Institute. At the same time he became the Director of the Bureau of Standards in the office in Berlin.

On the 12th of May 1856 at the 10th commemoration of the "Hutte", the renowned society of the pupils of the Royal Technical Institute, Grashof, along with Freidrich Karl Euler, was one of the leaders in founding the Society of German Engineers (*Vereines deutscher Ingenieure*, VDI). If the society was to endure it was decided that it must include the whole of Germany, and be the society of all German engineers. The true embodiment of the ideas of the Society became Franz Grashof. He had a well established reputation in the scientific world, participated as a Society director and was